Name: _____

Precal 11 – Ch. 4 Review

1. Factor the following:

a) $x^2 + 4x + 3$ ⊗ 3, ⊕ 4 → 3, 1

$(x+3)(x+1)$

b) $x^2 + 2x - 8$ ⊗ -8, ⊕ 2 → 4, -2

$(x+4)(x-2)$

c) $x^2 - x - 6$ ⊗ -6, ⊕ -1 → -3, 2

$(x-3)(x+2)$

d) $x^2 - 7x + 12$ ⊗ 12, ⊕ -7 → -4, -3

$(x-4)(x-3)$

e) $3x^2 + 10x + 7$ ⊗ 21, ⊕ 10 → 3, 7

$3x^2+3x+7x+7$

$3x(x+1)+7(x+1)$

$(3x+7)(x+1)$

f) $2x^2 - 9x + 9$ ⊗ 18, ⊕ -9 → -6, -3

$2x^2-6x-3x+9$

$2x(x-3)-3(x-3)$

$(2x-3)(x-3)$

g) $5(x-1)^2 + 12(x-1) + 7$

let $a = x-1$

$5a^2+12a+7$ ⊗ 35, ⊕ 12 → 7, 5

$5a^2+5a+7a+7$

$5a(a+1)+7(a+1)$

$(5a+7)(a+1)$

$(5(x-1)+7)(x-1+1)$

$(5x-5+7)(x)$

$(5x+2)(x)$

h) $169y^2 - 144x^2$ ← difference of squares

a^2 b^2

$(13y - 12x)(13y + 12x)$

2. Solve the following by factoring

a) $3x^2 - 11x + 10 = 0$ (×) 30 (+) -11 → -5, -6

$3x^2 - 6x - 5x + 10 = 0$

$3x(x-2) - 5(x-2) = 0$

$(3x-5)(x-2) = 0$

$3x = 5$ $x = 2$

$x = \frac{5}{3}$

b) $6x^2 - 13x - 5 = 0$ (×) -30 (+) -13 → -15, 2

$6x^2 + 2x - 15x - 5 = 0$

$2x(3x+1) - 5(3x+1) = 0$

$(2x-5)(3x+1) = 0$

$2x = 5$ $3x = -1$

$x = \frac{5}{2}$ $x = -\frac{1}{3}$

c) $5x^2 - 17x + 6 = 0$ (×) 30 (+) -17 → -15, -2

$5x^2 - 15x - 2x + 6 = 0$

$5x(x-3) - 2(x-3) = 0$

$(5x-2)(x-3) = 0$

$5x = 2$ $x = 3$

$x = \frac{2}{5}$

d) $3x^2 + 2x - 8 = 0$ (×) -24 (+) 2 → -4, 6

$3x^2 + 6x - 4x - 8 = 0$

$3x(x+2) - 4(x+2) = 0$

$(3x-4)(x+2) = 0$

$3x = 4$ $x = -2$

$x = \frac{4}{3}$

3. Solve the following by completing the square

a) $x^2 + 7x + 12 = 0$

$p = \frac{-7}{2(1)} = \frac{-7}{2}$

$q = 12 - (1)(-\frac{7}{2})^2$

$= -0.25$

$(x + \frac{7}{2})^2 - 0.25 = 0$

$(x + \frac{7}{2})^2 = 0.25$

$x + \frac{7}{2} = \pm\sqrt{0.25}$

$x = \pm\sqrt{0.25} - \frac{7}{2}$

b) $x^2 + 9x + 8 = 0$

$p = \frac{-9}{2(1)} = -\frac{9}{2}$

$q = 8 - (1)(-\frac{9}{2})^2$

$= -12.25$

$(x + \frac{9}{2})^2 - 12.25 = 0$

$(x + \frac{9}{2})^2 = 12.25$

$x + \frac{9}{2} = \pm\sqrt{12.25}$

$x = \pm\sqrt{12.25} - \frac{9}{2}$

d) $2x^2 - 15x + 8 = 0$

$p = \frac{+15}{2(2)} = \frac{15}{4}$

$q = 8 - (2)(\frac{15}{4})^2$

$= -20.125$ or $-\frac{161}{8}$

$2(x - \frac{15}{4})^2 - 20.125 = 0$

$2(x - \frac{15}{4})^2 = 20.125$

$x - \frac{15}{4} = \pm\sqrt{\frac{20.125}{2}}$

$x = \pm\sqrt{\frac{20.125}{2}} + \frac{15}{4}$

e) $-4x^2 + 7x - 9 = 0$

$p = \frac{-7}{2(-4)} = \frac{-7}{-8} = \frac{7}{8}$

$q = -9 - (-4)(\frac{7}{8})^2$

$= -\frac{95}{16}$

$-4(x - \frac{7}{8})^2 - \frac{95}{16} = 0$

$-4(x - \frac{7}{8})^2 = \frac{95}{16}$

$(x - \frac{7}{8})^2 = -\frac{95}{64}$

no solutions because can't square root $-\frac{95}{64}$

4. Solve the following using the quadratic formula

a) $2x^2 - 7x + 6 = 0$

$x = \frac{7 \pm \sqrt{(-7)^2 - 4(2)(6)}}{2(2)}$

$= \frac{7 \pm \sqrt{1}}{4}$

$x = 2, \ x = \frac{3}{2}$

b) $9x^2 - 8x - 1 = 0$

$x = \frac{8 \pm \sqrt{(-8)^2 - 4(9)(1)}}{2(9)}$

$= \frac{8 \pm \sqrt{28}}{18}$

$x = \frac{8 + \sqrt{28}}{18}, \frac{8 - \sqrt{28}}{18}$ OR 0.74, 0.15

c) $5x^2 - 17x + 6 = 0$

$x = \frac{+17 \pm \sqrt{(-17)^2 - 4(5)(6)}}{2(5)}$

$= \frac{17 \pm \sqrt{169}}{10}$

$x = 3, \frac{2}{5}$

d) $4x^2 + 12x + 5 = 0$

$x = \frac{-12 \pm \sqrt{(12)^2 - 4(4)(5)}}{2(4)}$

$= \frac{-12 \pm \sqrt{64}}{8}$

$= -\frac{1}{2}, -\frac{5}{2}$

5. Solve the following using the method of your choice

a) $(x - 7)^2 = (x + 3)^2$

$x^2 - 14x + 49 = x^2 + 6x + 9$

$-20x - 40 = 0$

$\frac{-20x}{-20} = \frac{40}{-20}$

$x = -2$

b) $\frac{1}{4}x^2 - 2x + 7 = 0$

$\frac{\frac{1}{4}(x^2 - 8x + 28) = 0}{\frac{1}{4}} \quad \frac{}{1/4}$

$x^2 - 8x + 28 = 0$

$x = \frac{8 \pm \sqrt{(-8)^2 - 4(1)(28)}}{2(1)}$ ← $\Delta < 0$

∴ no roots.

c) $25x^2 + 80x + 61 = 0$

$p = \frac{-80}{2(25)} = \frac{-80}{50} = -\frac{8}{5}$

$q = 61 - (25)(-\frac{8}{5})^2$

$= 61 - 64$

$= -3$

$25(x + \frac{8}{5})^2 - 3 = 0$

$\frac{25(x + 8/5)^2}{25} = \frac{3}{25}$

$(x + \frac{8}{5})^2 = 3/25$

$x + \frac{8}{5} = \pm\sqrt{3/25}$

$x = \pm\sqrt{3/25} - \frac{8}{5}$

d) $\frac{1}{49}x^2 - 81 = 0$ ← difference of squares

$(\frac{1}{7}x - 9)(\frac{1}{7}x + 9) = 0$

$\frac{1}{7}x = 9 \qquad \frac{1}{7}x = -9$

$x = 63 \qquad x = -63$

6. How many roots does the quadratic $-4x^2 + 68x - 120$ have? Use the discriminant.

$68^2 - 4(-4)(-120)$

$= 4624 - 1920$

$= 2704$

$\therefore$ there are 2 roots

7. What values of k in the quadratic $4x^2 - 13x + k$ yield

a) one real root

$(-13)^2 - 4(4)(k) = 0$

$169 - 16k = 0$

$\frac{169}{16} = \frac{16k}{16}$

$\boxed{\frac{169}{16} = k}$

b) two real roots

$169 - 16k > 0$

$\frac{169}{16} > \frac{16k}{16}$

$\boxed{\frac{169}{16} > k}$

c) no roots

$169 - 16k < 0$

$\frac{169}{16} < \frac{16k}{16}$

$\boxed{\frac{169}{16} < k}$

8. The difference between the squares of two numbers is 75. One number is double the other. What is the product of the two numbers?

let x: 1st # let y: 2nd #.

$x^2 - y^2 = 75$

$x = 2y$

$(2y)^2 - y^2 = 75$

$4y^2 - y^2 = 75$

$3y^2 = 75$

$y^2 = \frac{75}{3}$

$y^2 = 25$

$y = \pm 5$

9. A ball is thrown upwards from a rooftop, 80 m above the ground. It will reach a maximum vertical height and then fall back to the ground. The height of the ball from the ground at time t is h, which is given by the function

$h(t) = -16t^2 + 64t + 80$

use graphing calculator

a) What is the maximum height the ball reaches?

vertex (2, 144)

∴ max height is 144m

b) How long will it take for the ball to reach the ground?

x-intercept at (5, 0)

∴ it takes 5 seconds.

c) What is the height of the ball at t=0?

80 m

d) How high is the ball after 4 seconds?

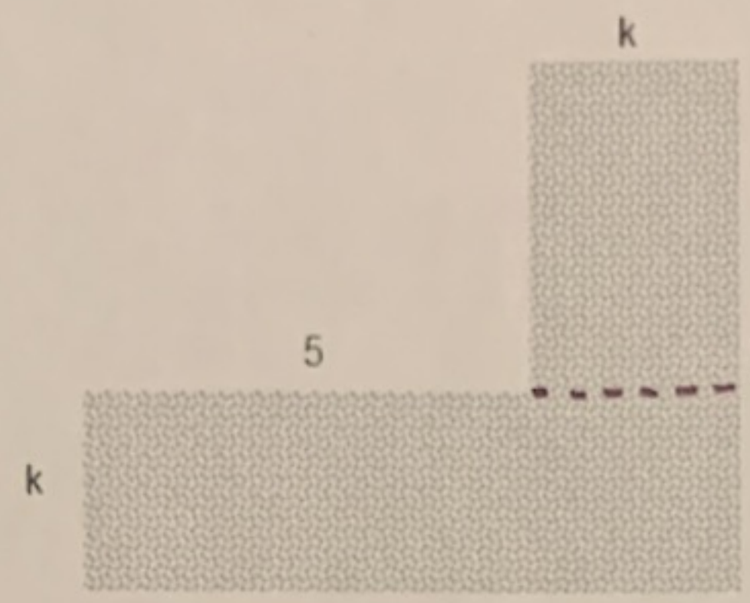
80 m

e) What are the domain and range of this situation?

$0 \leq x \leq 5$

$0 \leq y \leq 80$

10. The following picture shows the shape of a certain grass patch. If the area of the patch is $80m^2$, find k



← Area = $k(2k+1-k)$
$= k(k+1)$
$= k^2+k$

← Area = $k(5+k)$
$= 5k+k^2$

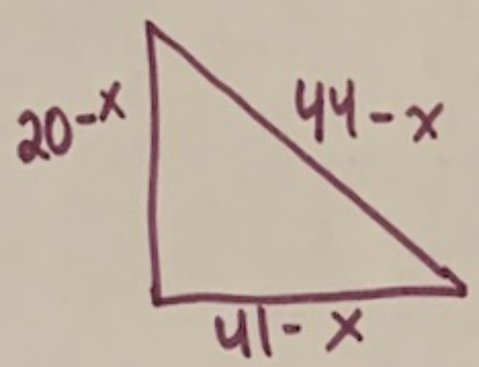
Total Area = $(5k+k^2)+(k^2+k)$
$= 2k^2+6k$

$80 = 2k^2+6k$
$0 = 2k^2+6k-80$
$0 = 2k^2+10k-16k-80$
$0 = 2k(k+5)-16(k+5)$

→ $0 = (2k-16)(k+5)$
$2k = 16$ $k = -5$
$k = 8$ ↑ reject.
$\therefore k = 8m$

11. Three rods measure 20 cm, 41 cm and 44 cm. If the same length is cut off each piece, the remaining lengths can be formed into a right triangle. What length is cut off? (HINT: recall the Pythagorean theorem: $a^2 + b^2 = c^2$)

let x = length that was cut off



← we know $44-x$ must be on the hypotenuse because it is the longest length.

$(20-x)^2 + (41-x)^2 = (44-x)^2$
$400 - 40x + x^2 + 1681 - 82x + x^2 = 1936 - 88x + x^2$

$x^2 - 34x + 145 = 0$
$(x-29)(x-5) = 0$
$x = 29$ $x = 5$
reject ↑ b/c too big.

$\therefore$ 5cm was cut off.