Name: ______________________

Precal 11 – Ch. 3 Review

1. How can you tell whether a function is a quadratic or not if you aren't given its graph?

a quadratic will have an x^2 term and no larger degree (i.e. no x^4 or x^3).

2. What is another word for "zero" of a function?

root, x-intercept, solution.

3. If the vertex is $(2, -3)$, what is a **possible** formula for the quadratic? (hint: there are many different answers here)

$(2, -3) \rightarrow y = a(x-2)^2 - 3$

p q

One possible formula/function is $y = (x-2)^2 - 3$.

4. What is the equation for the axis of symmetry in the question above?

$x = 2$

5. In the quadratic you formed in question 3, is it narrower or wider than the graph of $y = x^2$?

It is neither since $a = 1$.

If $a > 1$, the graph would be narrower (same if $a < -1$). If $-1 < a < 1$, the graph would be wider

6. How does the graph of $y = x^2 - 5$ look compared to the graph of $y = x^2$?

shifted 5 units downwards

7. How does the graph of $y = (x - 3)^2$ look compared to the graph of $y = x^2$?

shifted to the right by 3.

8. How does the graph of $y = \frac{1}{2}(x - 2)^2 + 1$ look compared to the graph of $y = x^2$?

wider by a factor of $\frac{1}{2}$, shifted horizontally to the right by 2 units and shifted one unit up.

9. Given a quadratic with the vertex (2,2) and passing through the point (1,0), what is the equation for the quadratic? What is the y coordinate when $x = 3$?

$(2,2) \longrightarrow y = a(x-2)^2 + 2$

p q

$0 = a(1-2)^2 + 2$

$0 = a(-1)^2 + 2$

$0 = a + 2$

$-2 = a$

$\longrightarrow y = -2(x-2)^2 + 2.$

when $x = 3$: $y = -2(3-2)^2 + 2$

$y = -2(1) + 2$

$\boxed{y = 0.}$

10. What value of k will make $x^2 + 8x + k$ a perfect square trinomial?

$\left(\frac{8}{2}\right)^2 = (4)^2 = 16.$

$K = 16.$

11. Convert $y = x^2 + 10x + 24$ to vertex form.

$p = \frac{-10}{2(1)} = -\frac{10}{2} = -5$

$q = 24 - (1)(-5)^2$

$= 24 - 25$

$= -1$

$y = (x+5)^2 - 1$

12. Convert $y = -4x^2 + 15x - 36$ to vertex form

$p = \frac{-15}{2(-4)} = \frac{-15}{-8} = \frac{15}{8}$

$q = -36 - (-4)\left(\frac{15}{8}\right)^2$

$= -36 + (4)\left(\frac{225}{64}\right)$

$\longrightarrow = -36 + \frac{225}{16}$

$= \frac{-351}{16}$

$\therefore y = -4\left(x - \frac{15}{8}\right)^2 - \frac{351}{16}$

13. If the coordinate (3,9) lies on the graph of $y = x^2$, what will be the coordinates of this point on the graph of $y = -3(x+4)^2 - 5$?

x	y	
0	0	
1	1	>+1 x-3
2	4	>+3 x-3
3	9	>+5 x-3

$a = -3 \Longrightarrow$

x	y
0	0
1	-3
2	-12
3	-18

$q = -5 \Longrightarrow$

x	y
0	-5
1	-8
2	-17
3	-23

$p = -4 \Longrightarrow$

x	y
-4	-5
-3	-8
-2	-17
-1	-23

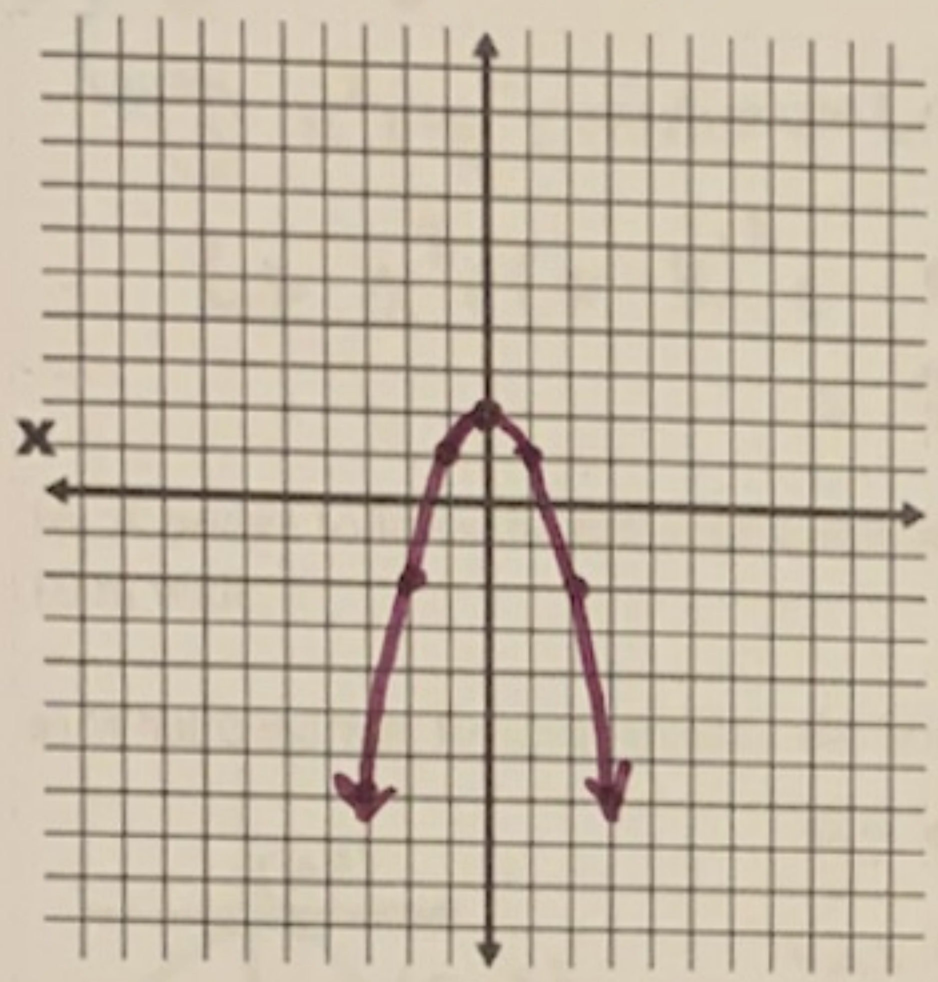
$\therefore$ the point will have coordinates $(-1, -23)$.

14. Graph the following parabolas without a graphing calculator. State the information required:

a) $y = -x^2 + 2$

$p = 0$ $a = -1$

$q = 2$



• plot vertex
• since $a = -1$, no wider or narrower so over 1 down 1, over 1 down 3, over 1 down 5...

Vertex: $(0,2)$

AOS: $x = 0$

y-intercept: $y = 2$

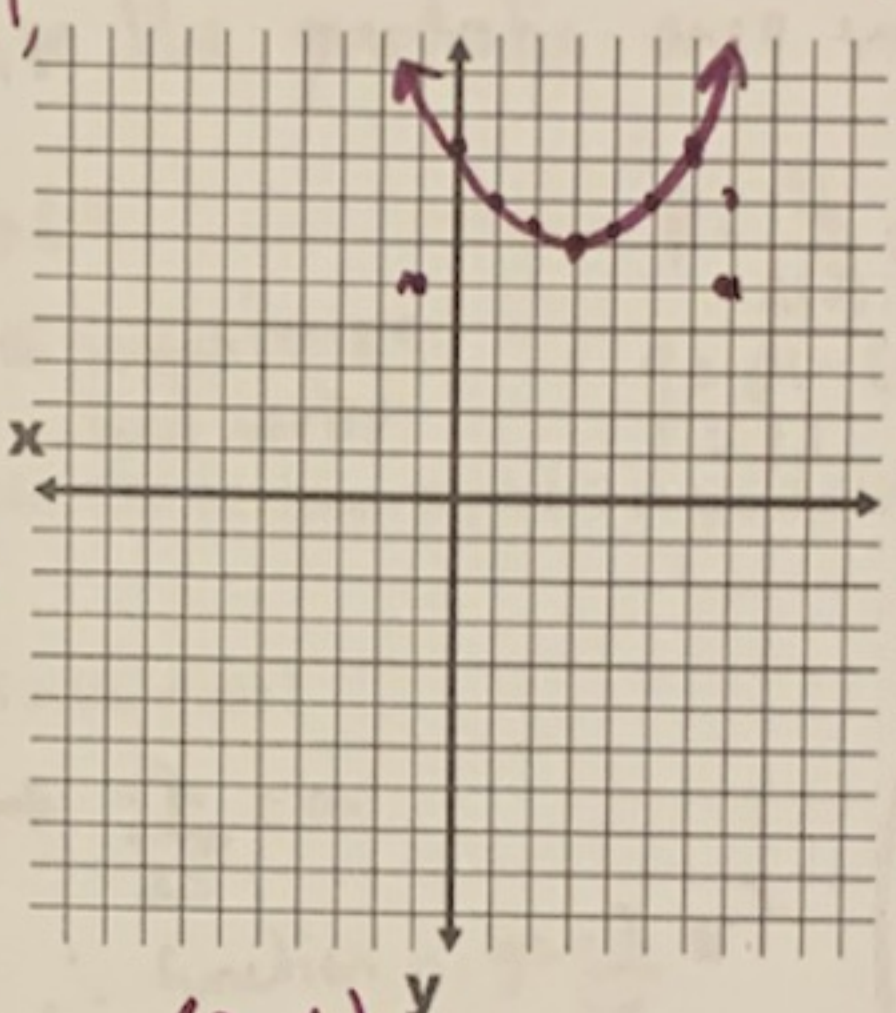
Domain: $x \in \mathbb{R}$

Range: $y \leq 2$

b) $y = \frac{1}{4}(x-3)^2 + 6$

$p = 3$ $q = 6$

$a = \frac{1}{4}$



x	y
0	0
1	1
2	4
3	9

x	y
0	0
1	1/4
2	1
3	9/4

x	y
3	6
4	6.25
5	7
6	8.25

Vertex: $(3,6)$

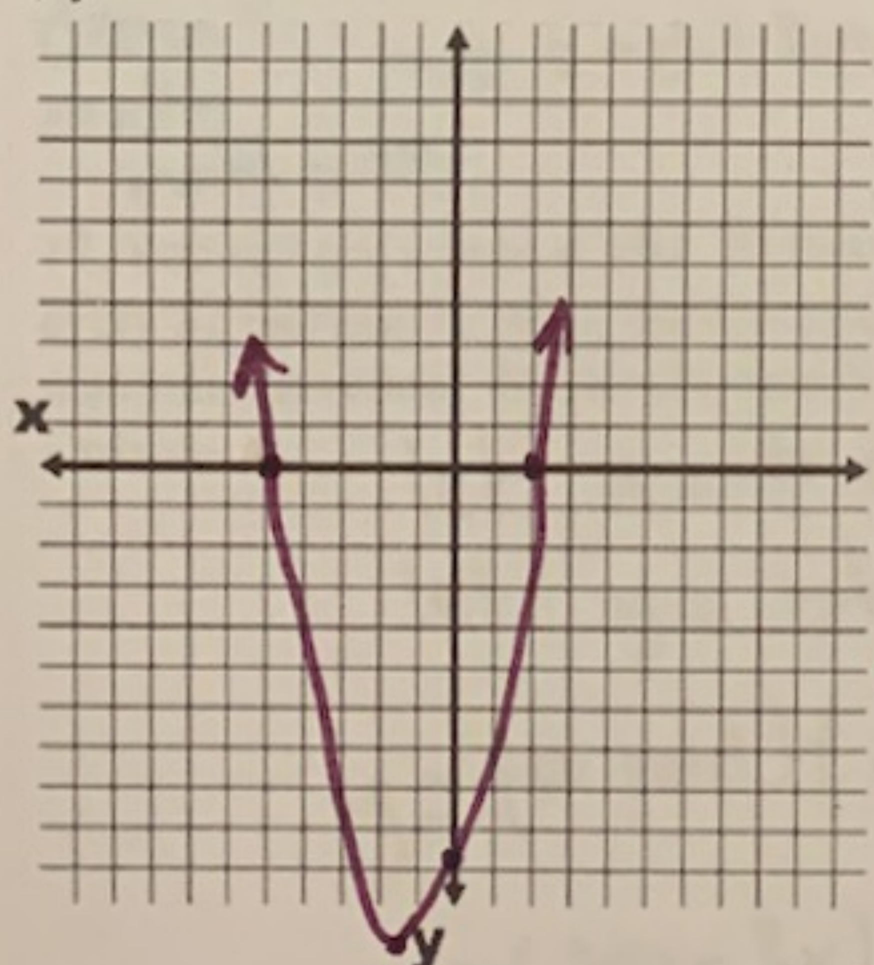
AOS: $x = 3$

y-intercept: ~~[illegible]~~ $y = 8.25$

Domain: $x \in \mathbb{R}$

Range: $y \geq 6$

c) $y = x^2 + 3x - 10$



$p = \frac{-3}{2(1)} = \frac{-3}{2}$

$q = -10 - (1)(\frac{3}{2})^2$

$= -10 - \frac{9}{4}$

$= \frac{-49}{4}$

roots: $(x+5)(x-2)$

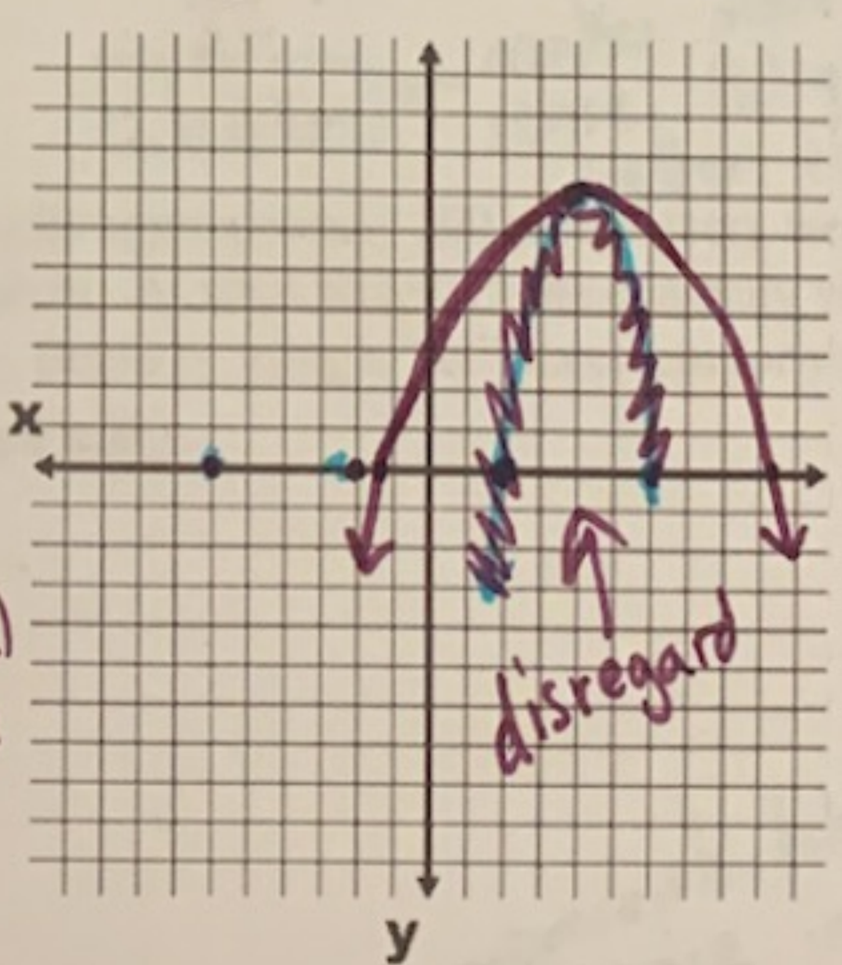
$x = -5$ $x = 2$

Vertex: $(\frac{-3}{2}, \frac{-49}{4})$ Domain: $x \in \mathbb{R}$

AOS: $x = -3/2$ Range: $y \geq -49/4$

y-intercept: $y = -10$

d) $y = -x^2 + 8x + 12$



$p = \frac{-8}{2(-1)} = 4$

$q = 12 - (-1)(4)^2$

$= 12 + 16$

$= 28$

roots: ~~[illegible]~~

$\underbrace{x = -1.29, 9.29}_{\text{to find, use quadratic formula}}$

Vertex: $(4, 28)$ Domain: $x \in \mathbb{R}$

AOS: $x = 4$ Range: $y \leq 28$

y-intercept: $y = 12$

① ②

15. Two numbers have a difference of 8 and the sum of their squares is a minimum. Determine the two numbers.

let x = first #, y = 2nd #.

sub into ② to eliminate y.

① $x - y = 8 \Rightarrow x - 8 = y$

② $x^2 + y^2$ = minimum (must minimize the quadratic once we find it.)

$\hookrightarrow x^2 + (x-8)^2 = x^2 + x^2 - 16x + 64$

$= 2x^2 - 16x + 64$ ← to minimize, find vertex:

$p = \frac{16}{2(2)} = 4$

$q = 64 - (2)(16)$
$= 32$

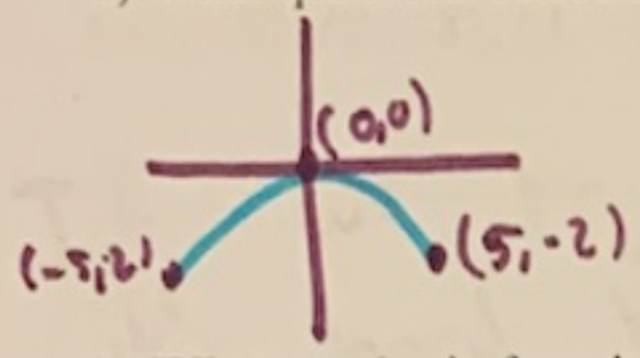
∴ vertex is (4, 32)

value of 1st #

$4 - 8 = y$
$-4 = y$
∴ the 2 #s are −4 and 4

16. A bridge follows the shape of a downward facing parabola. The maximum depth is 2 m and the bridge is 10 m wide.

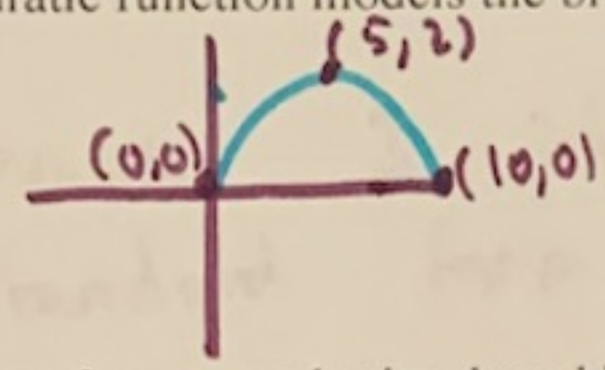
a) What quadratic function models the bridge if the vertex is at the origin?



$y = a(x-0)^2 + 0$
$-2 = a(5)^2 + 0$
$\frac{-2}{25} = \frac{25a}{25}$
$\frac{-2}{25} = a$

∴ function is $y = \frac{-2}{25}x^2$

b) What quadratic function models the bridge if the farthest left side is on the y-axis?



$y = a(x-5)^2 + 2$
$0 = a(10-5)^2 + 2$
$0 = 25a + 2$
$-2 = 25a$
$\frac{-2}{25} = a$

∴ function is $y = \frac{-2}{25}(x-5)^2 + 2$

c) Using either of your quadratics, how high off the ground is a person standing 2 m onto the bridge?

$y(2) = \frac{-2}{25}(2)^2$

$= -0.32$ m

doesn't really make sense.

$y(2) = \frac{-2}{25}(2-5)^2 + 2$

$= \frac{-2}{25}(9) + 2$

$\boxed{= 1.28\text{m}}$ ← makes more sense. (b) quadratic is a better representation of the situation.

17. You sell toy robots at $80. At this price, the company sells approximately 200 robots every week. For every $2 increase in price, the company will sell 3 fewer robots. At what price will the company yield the maximum revenue? Model this situation using a quadratic. Begin by using a "let" statement for your variables.

let x = # of price increases.
let y = total revenue.

y = (# sold)(cost/item)

$y = (200 - 3x)(80 + 2x)$
$y = 16000 + 400x - 240x - 6x^2$
$y = -6x^2 + 160x + 16000$

$p = \frac{-160}{2(-6)} = \frac{40}{3}$

$q = 16000 - (-6)(\frac{40}{3})^2$
$= 16000 + (6)(\frac{1600}{9})$
$= 17066.67$

∴ vertex is $(\frac{40}{3}, 17066.67)$.

represents total # of price increases

represents max revenue.

so if you increase the price $\frac{40}{3}$ or 13.33 times, you will create the max revenue. ∴ the robot should cost $80 + 2(13.3) = \$106.66$

18. You have 100 m of fencing. You need to fence off 3 sides of a rectangle because your property is beside a lake. What is the maximum area of the property you are fencing?

$P = 2x + y$

$100 = 2x + y$

$100 - 2x = y$

$A = x \cdot y$

$A = x(100 - 2x)$

$A = 100x - 2x^2$

$A = -2x^2 + 100x$ ← find max

$p = \frac{-100}{-2(2)} = \frac{100}{4} = 25$

$q = 0 - (-2)(25^2) = 1250$

∴ vertex is $(25, 1250)$ ↑ max area

∴ max area is $1250 m^2$.

19. Given the following points, find the quadratic equation in standard form that goes through these 3 points: $(-1, -9)$, $(0, -3)$, $(3, -33)$.

If we try and sketch this, we'll see that it's hard to determine what the vertex is.

We have 3 points to use and we must write it in standard form. ∴ we can create the following 3 equations:

$-9 = a(-1)^2 + b(-1) + c \longrightarrow -9 = a - b + c$

$-3 = a(0)^2 + b(0) + c \longrightarrow \therefore -3 = c$

$-33 = a(3)^2 + b(3) + c \longrightarrow -33 = 9a + 3b + c$

we have 3 equations and 3 unknowns: that's enough info to solve for a, b & c:

$-9 = a - b - 3 \longrightarrow (-6 = a - b)3 \Longrightarrow -18 = 3a - 3b$

$-33 = 9a + 3b - 3 \qquad -30 = 9a + 3b \qquad + \; -30 = 9a + 3b$

$-48 = 12a \quad 0$

$\frac{-48}{12} = \frac{12a}{12}$

$-4 = a$

$-9 = -4 - b - 3$

$-9 = -7 - b$

$-2 = -b$

$\frac{-2}{-1} = \frac{-b}{-1}$

$2 = b$

∴ equation is $\boxed{y = -4x^2 + 2x - 3}$

This is a challenging question & calls back your understanding of solving systems of equations from precal 10.