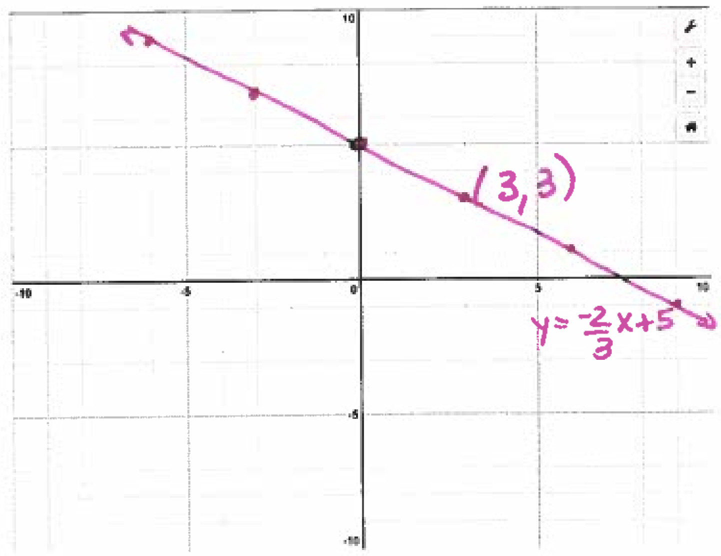
Vertex Form in Quadratic Functions

Key

Review from grade 10:

Linear Functions are straight lines.

$y = -\frac{2}{3}x + 5$



Slope-intercept form: $y = mx + b$ — y-int

slope

$y = -\frac{2}{3}x + 5$

$m = -\frac{2}{3}$ $b = 5$

General Form: $Ax + By + C$

$3[y = -\frac{2}{3}x + 5]$

$3y = -2x + 15$

$\boxed{2x + 3y - 15 = 0.}$

Slope-point form: $y - y_1 = m(x - x_1)$

$y - y_1 = -\frac{2}{3}(x - x_1)$

$\boxed{y - 3 = -\frac{2}{3}(x - 3)}$

Definitions:

Quadratic Function: a function f whose value $f(x)$ is given by a polynomial of degree 2. All quadratic functions graph a **parabola**.

Function Notation of $y = x^2$

$f(x) = x^2$

Circle the quadratic functions:

$y = x^2 + 2x + 1$	$y = 5x^3 + x^2$
$y = x^2$	$y = x^2 + \frac{1}{x}$
$y = 3x^2 + 2x^{\frac{1}{2}}$	$y = x^2 + x^{-1}$

Vertex: the maximum or minimum point of a graph; written as a coordinate (p,q) (x,y)

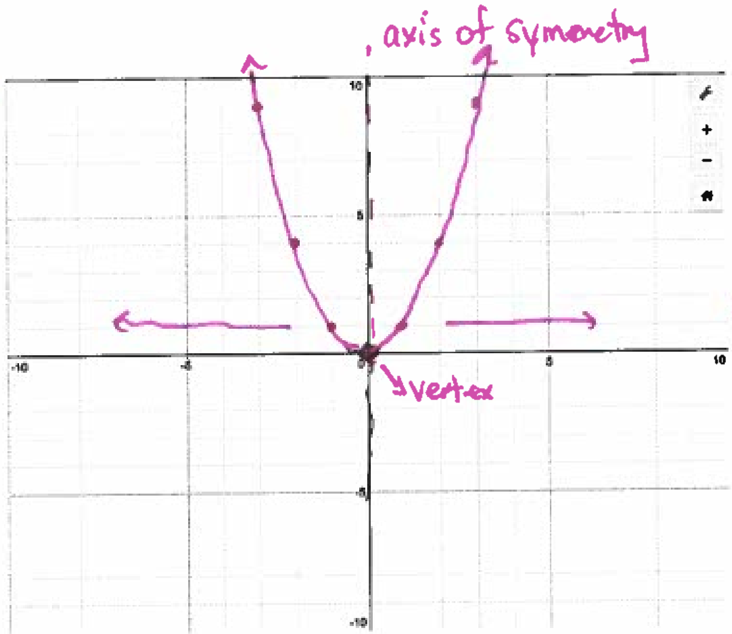
Axis of symmetry: the equation of a line which intersects the vertex of the graph and divides the graph into two equal halves; written as an equation x = p.

Domain: all possible values of x.

~~all real number~~

Range: all possible values of y.

x	y
-3	9
-2	4
-1	1
0	0
1	1
2	4
3	9



The simplest form of a quadratic is
$f(x) = x^2$ → $y = x^2$ $y = (-3)^2 = 9$

Vertex: (p, q) $(0,0)$

Axis of Symmetry: $x = p$ $\boxed{x = 0}$

Minimum point or Maximum point: $y = q$ $\boxed{y = 0}$

Domain: $x \in \mathbb{R}$

Range: For parabolas opening up $y \geq$ min value
For parabolas opening down $y \leq$ max value

Vertex Form of a Quadratic Function is $f(x) = a(x-p)^2 + q$

a = direction of opening, width of parabola ~~direction of~~

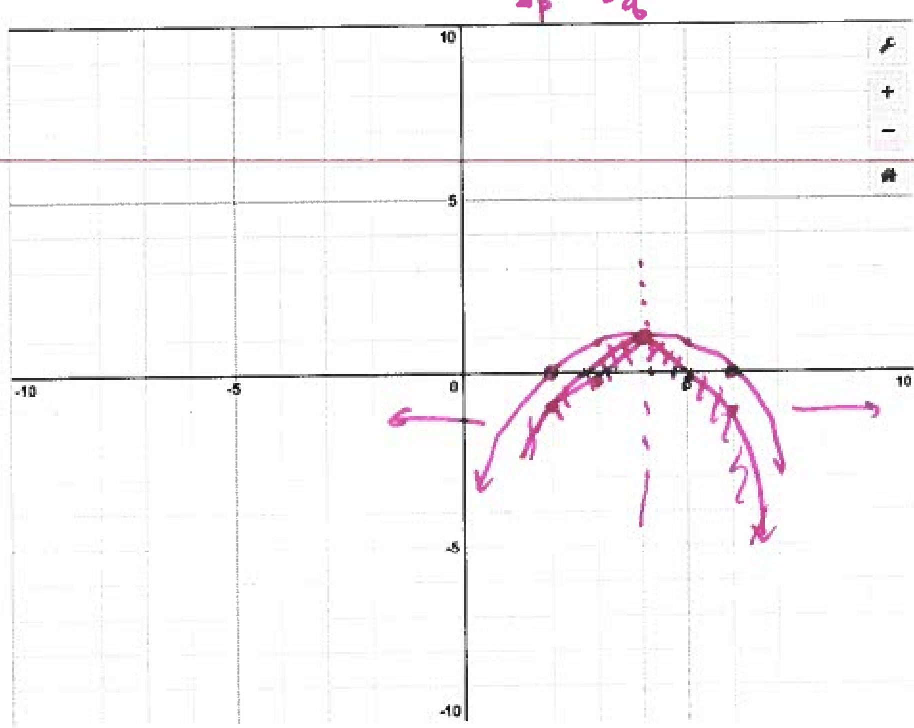
p: moves the parabola left or right

q: moves the parabola up or down.

Ex. 1) Determine the vertex, domain and range, direction of opening, equation of the axis of symmetry, then sketch each graph for the following function.

$y = -\frac{1}{4}(x-4)^2 + 1$ (p = 4, q = 1)

x	y
3	3/4
4	1
5	3/4
6	0



Direction of opening: down

Vertex: (4, 1)

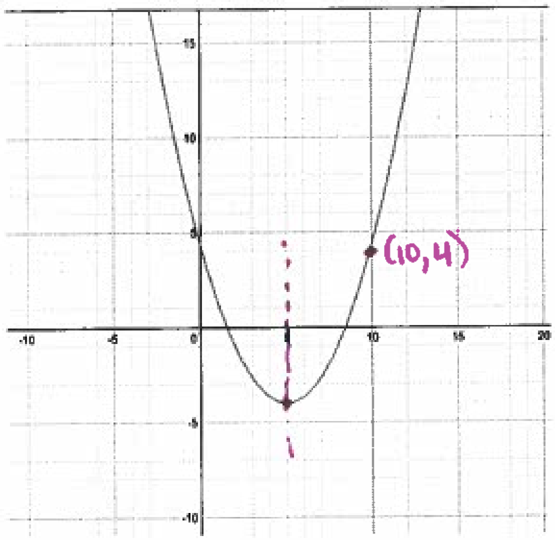
Equation of axis of symmetry: x = 4

Domain: $x \in \mathbb{R}$

Range: $y \leq 1$

maximum point @ y = 1

Ex 2) Determine a quadratic function in vertex form for the graph.



vertex is (5, -4)
(p, q)

vertex form $y = a(x-p)^2 + q$

$y = a(x-5)^2 \cancel{+} - 4$

$4 = a(10-5)^2 - 4$

$4 = 25a - 4$

$+4 \qquad + 4$

$\frac{8}{25} = \frac{25a}{25}$

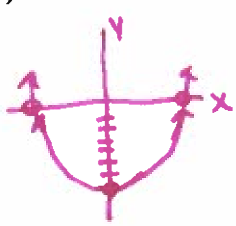
$8/25 = a$

$\boxed{y = \frac{8}{25}(x-5)^2 + 10}$

Ex 3) Determine the number x-intercepts for each quadratic function without graphing.

$y = a(x-p)^2 + q$

a) $f(x) = 0.5x^2 - 7$ vertex: (0, -7)

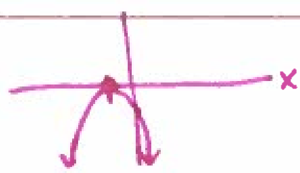


two x-intercepts One y-intercept

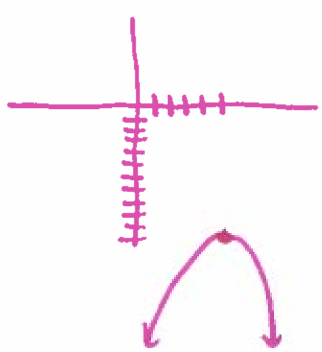
b) $f(x) = -2(x+1)^2$ vertex : (-1, 0)

$y = -2[x-(-1)]^2$

one x-intercept one y-intercept

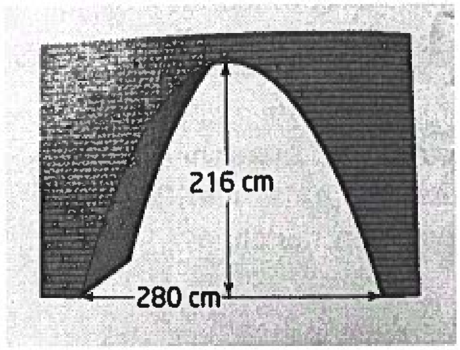


c) $f(x) = -\frac{1}{6}(x-5)^2 - 11$ vertex (5, -11)

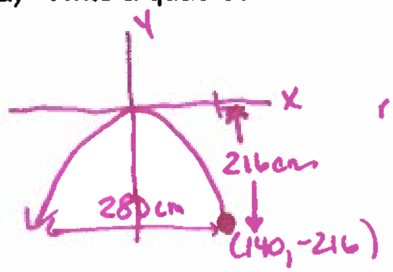


No x-intercept one y-intercept.

Ex 4) Suppose a parabolic archway has a width of 280 cm and a height of 216 cm at its highest point above the floor.



a) Write a quadratic function in vertex form that models the shape of the archway.



$y = ax^2$

$-216 = a(140)^2$

$\frac{-216}{19600} = \frac{19600a}{19600}$

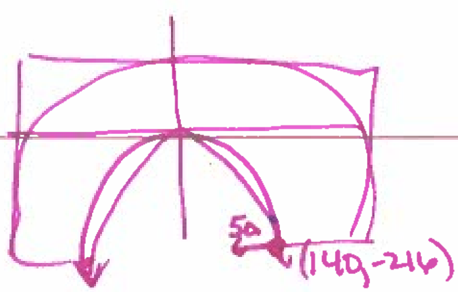
$\frac{-27}{2450} = a$

$\frac{-27}{2450} = a$

$= a$

$y = \frac{-27}{2450}x^2$

b) Determine the height of the archway at a point that is 50 cm from its outer edge.



$x = 90$ cm

$f(x) = \frac{-27}{2450}x^2$

$f(90) = \frac{-27}{2450}(90)^2$

$= \frac{-27(8100)}{2450}$

$= -89.27$ cm 89.27 cm

The height of the archway at a point that is 50 cm from its outer edge is 89.27 cm

Pg. 157 # 4, 7 – 9, 12, 15, 16, 18, 20, 21.