The **standard form** of a quadratic function is $f(x) = ax^2 + bx + c$ where a, b, and c are real numbers with $a \neq 0$

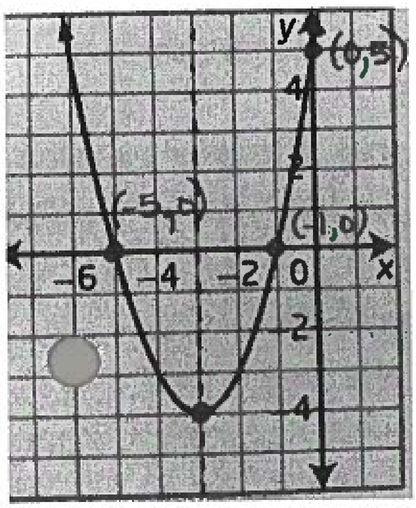
- a determines the shape and whether the graph opens upward or downward.
- b influences the position of the graph.
- c determines the y-intercept of the graph

Deriving the standard form of a quadratic equation:

$f(x) = a(x-p)^2 + q$

$f(x) = a(x-p)(x-p) + q$
$= a(x^2 - xp - xp + p^2) + q$
$= a(x^2 - 2xp + p^2) + q$
$= ax^2 - 2apx + (ap^2 + q)$
$= ax^2 + bx + c$
$b = -2ap$ OR $p = \frac{b}{-2a}$
$c = ap^2 + q$ OR $q = c - ap^2$

Ex 1) For the quadratic function, $y = x^2 + 6x + 5$, determine the following:



$y = x^2 + 6x + 5$
$p = \frac{b}{-2a} = \frac{6}{-2} = -3$
$q = c - ap^2$
$q = 5 - 1(-3)^2$
$q = 5 - 9 = -4$

Direction of opening: upwards

Coordinates of the vertex: $(-3, -4)$

Maximum or minimum value: min @ -4

Equation of the axis of symmetry: $x = p$, $x = -3$

x-intercepts: $(-5, 0)$, $(-1, 0)$

y-intercept: $(0, 5)$

Domain: $x \in \mathbb{R}$

Range: $y \geq -4$

y-intercept, $x = 0$

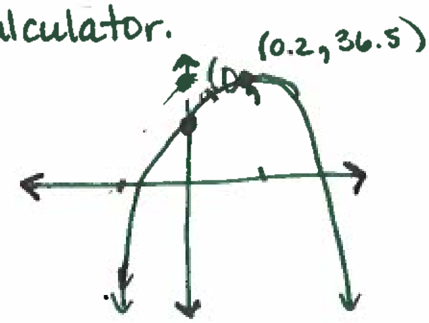
$y = x^2 + 6x + 5$
$y = 0^2 + 6(0) + 5$
$y = 5$
$(0, 5)$

Ex 2) A frog sitting on a rock jumps into a pond. The height, h, in centimetres, of the frog above the surface of the water as a function of time, t, in seconds, since it jumped can be modelled by the function $h(t) = -490t^2 + 150t + 25$. Where appropriate, answer the following questions to the nearest tenth.

opens downwards

y-intercept.

a) Graph the function. Use a graphing calculator.



b) What is the y-intercept? What does it represent in this situation?

(0, 25) After 0 seconds, the height of the frog is 25 cm.

c) What maximum height does the frog reach? When does it reach that height?

2nd → calc → max → enter
Leftbound → enter
Rightbound enter
Guess? enter

max @ y = 36.5 cm
x = 0.2 seconds.

The frog reaches a maximum height ~~after~~ at 0.2s.

d) When does the frog hit the surface of the water?

Height @ the surface is 0 cm, where y = 0. On our graph it is the positive x-intercept.

x = 0.43 when y = 0.

The frog hits the surface of the water after 0.4 seconds.

e) What are the domain and range in this situation?

domain: independent variable $\{t \mid 0 \le t \le 0.4, t \in \mathbb{R}\}$
range: $\{h \mid 0 \le h \le 36.5, h \in \mathbb{R}\}$

f) How high is the frog 0.25 s after it jumps?

~~2nd → graph → table~~

Trace till you find x = 0.25 s. y ≈ 31.4 cm.

By Substitution

$h(t) = -490t^2 + 150t + 25$

$h(0.25) = -490(0.25)^2 + 150(0.25) + 25$

$= -30.625 + 37.5 + 25$

$= 31.875$

Ex 3) At a children's music festival, the organizers are roping off a rectangular area for stroller parking. There is 160 m of rope available to create the perimeter.

$A = l \times w$
$= (80 - w)w = 80w - w^2$

a) Write a quadratic function in standard form to represent the area for the stoller parking.

l

w

$P = 2l + 2w$

$2l + 2w = 160 \rightarrow$ solve for l

$A = l \times w$

$2l + 2w = 160$
$-2w \quad -2w$
$\frac{2l}{2} = \frac{160}{2} - \frac{2w}{2}$

$l = 80 - w$

b) What are the coordinates of the vertex? What does the vertex represent in this situation?

$x = \frac{-b}{2a}$
$= \frac{-80}{2(-1)} = 40$

$A = 80w - w^2$
$A = 80(40) - (40)^2$
$A = 3200 - 1600$
$= 1600$

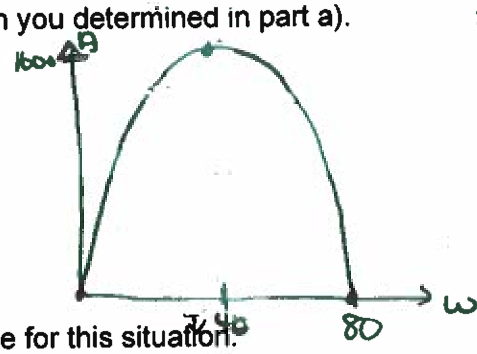
vertex: $(40, 1600)$

c) Sketch the graph for the function you determined in part a).

2nd $\rightarrow$ calc $\rightarrow$ 2 (zero)

Go to table to find more points.

Change window:
Xmin: -10
Xmax: 100
Ymin: -10
Ymax: 1700



d) Determine the domain and range for this situation.

Domain: $\{w \mid 0 \le w \le 80, w \in \mathbb{R}\}$

Range: $\{A \mid 0 \le A \le 1600, A \in \mathbb{R}\}$

e) Identify any assumptions you made.

We assumed that the whole 160m of rope will be used. Also, any length or width between 0m to 80m is possible. We don't know the available workable area ~~that~~ in the parking lot.

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