Review

**To convert from vertex form to standard form we expand and simplify:**

$y = 5(x-3)^2 - 38$ **(vertex form)**

$y = 5(x-3)(x-3) - 38$
$= 5(x^2 - 3x - 3x + 9) - 38$
$= 5(x^2 - 6x + 9) - 38$
$= 5x^2 - 30x + 45 - 38$
$= 5x^2 - 30x + 7$ (Standard form)

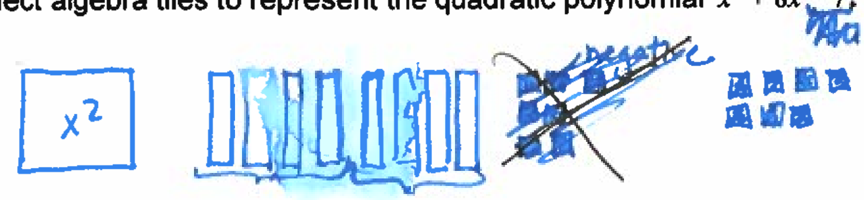
Ex 1) Rewrite each function in vertex form by completing the square.

a) $f(x) = x^2 + 8x - 7$ when a = 1

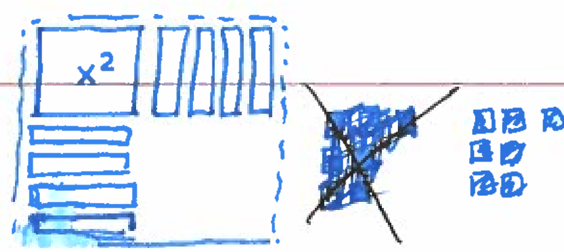
$a = 1 \quad b = 8 \quad c = -7$

**Model with algebra tiles:**

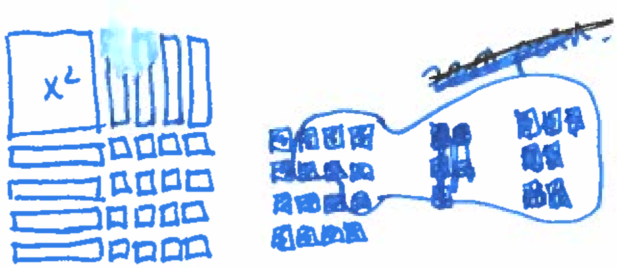
Select algebra tiles to represent the quadratic polynomial $x^2 + 8x - 7$.




Using the $x^2$-tile and $x$-tile, create an incomplete square to represent the first two terms. Leave the unit tiles aside for now.




To complete the square, add sixteen zero pairs. The sixteen positive unit tiles complete the square and the sixteen negative unit tiles are necessary to maintain an expression equivalent to the original.



Simplify the expression by removing zero pairs.


You can express the completed square in the expanded form as $x^2+8x+16$, but also as the square of a binomial as $(x+4)^2$. The vertex form of the function is

$$\boxed{y=(x+4)^2-23}$$

**Use an Algebraic Method:**

$y=x^2+8x-7$

$y=(x^2+8x)-7$ **Group the first two terms**

$\frac{8}{2}=4^2=16$ $\quad y=(x^2+8x+16-16)-7$ **Inside the brackets, add and subtract the Square of half the coefficient of the x-term**

$y=(x^2+8x+16)-16-7$ **Group the perfect square trinomial**

$=$

$y=(x+4)^2-16-7$ **Rewrite the perfect square trinomial as the Square of a binomial**

$\boxed{y=(x+4)^2-23}$ **Simplify**

b) $y=-3x^2-18x-24$, when a > 1

$a=-3$ $\quad b=-18$ $\quad c=-24$

~~y=-3(x²~~ $y=(-3x^2-18x)-24$ Group the first two terms

$y=-3(x^2-6x)-24$ Factor out the leading coefficient

$-\frac{6}{2}=(-3)^2=9$ $\quad y=-3[(x^2-6x+9-9)]-24$ inside the brackets, add and subtract the square of half of the coefficient of the x-term

$y=-3(x^2-6x+9)+$[illegible]$-24$ Group the perfect square trinomial

$y=-3(x-3)^2+3$ Rewrite the perfect square as the square of a binomial

Expand the square brackets and simplify.

c) $y = 2x^2 - 20x$, when c = 0

$y = 2(x^2 - 10x)$

$\frac{-10}{2} = (-5)^2 = 25$  $y = 2[(x^2 - 10x + 25) - 25)]$

$y = 2[(x^2 - 10x + 25) - 25]$

$= 2(x^2 - 10x + 25) - 50$

$= 2(x - 5)^2 - 50$

Ex 2) a) Convert the function $y = -3x^2 - 27x + 13$ to vertex form.

$y = (-3x^2 - 27x) + 13$

$y = -3(x^2 - 9x) + 13$

$\left(\frac{-9}{2}\right)^2 = \frac{81}{4}$  $= -3\left[\left(x^2 - 9x + \frac{81}{4}\right) - \frac{81}{4}\right] + 13$

$= -3\left[\left(x - \frac{9}{2}\right)^2 - \frac{81}{4}\right] + 13$

$= -3\left(x - \frac{9}{2}\right)^2 + \frac{243}{4} + 13$

$\boxed{y = -3\left(x - \frac{9}{2}\right)^2 + \frac{295}{4}}$

b) Verify that the two forms are equivalent.

① Work backward

$y = -3(x - 4.5)^2 + 73.75$

$y = -3(x - 4.5)(x - 4.5) + 73.75$

$= -3(x^2 - 9x + 20.25) + 73.75$

$= -3x^2 + 27x - \cancel{410.0625}\ 60.75 + 73.75$

$\boxed{y = -3x^2 + 27x + 13}$

② Technology

Graph together. and you should only see the one graph

Ex 3) Consider the function $y = 3x^2 + 30x + 41$

a) Complete the square to determine the vertex of the graph of the function.

$y = (3x^2 + 30x) + 41$

$= 3(x^2 + 10x) + 41$

$\left(\frac{10}{2}\right)^2 = (5)^2 = 25$ $= 3(x^2 + 10x + 25 - 25) + 41$

$= 3[(x+5)^2 - 25] + 41$

$= 3(x+5)^2 - 75 + 41$

$= 3(x+5)^2 - 34$

b) Use $x = -\frac{b}{2a}$ and the standard form of the quadratic function to determine the vertex. Compare with your answer from part a).

$a = 3$ $b = 30$ $c = 41$

$x = -\frac{30}{2(3)} = -5$ same as (a).

Ex 4) A sporting goods store sells reusable sports water bottles for $8. At this price their weekly sales are approximately 100 items. Research says that for every $2 increase in price, the manager can expect the store to sell five fewer water bottles.

a) Represent this situation with a quadratic function.

Revenue = (price)(number of ~~sports~~ water bottles)

~~$y = (8x)(2x-5)$~~

~~$y = 16x^2 - 40x$~~

$y = (8 + 2n)(100 - 5n)$

$= 800 - 40n + 200n - 10n^2$

$800 + 160n - 10n^2$

b) Determine the maximum revenue the manager can expect based on these estimates. What selling price will give that maximum revenue?

$y = -10n^2 + 160n + 800$

$y = -10(n^2 - 16n) + 800$

$= -10(n^2 - 16n + 64 - 64) + 800$

$= -10(n-8)^2 + 640 + 800$

$= -10(n-8)^2 + 1440.$

max. revenue = $1440.

Price per bottle should be $8 + 2(8) = \$24$

c) Verify your solution.

Graph.

d) Explain any assumptions you made in using a quadratic function in this situation.

That price affects the revenue.

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