

## 4.4 Quadratic Formula

We can solve quadratic equations in standard form,  $ax^2 + bx + c = 0$ ,  $a \neq 0$ , using the quadratic formula,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\text{discriminant} = b^2 - 4ac$$

Ex 1) Use the discriminant to determine the nature of the roots for each quadratic equation.

a)  $x^2 - 5x + 4 = 0$   
 $a=1$   $b=-5$   $c=4$

$$b^2 - 4ac$$

$$(-5)^2 - 4(1)(4) = 25 - 16 = 9$$

if  $b^2 - 4ac > 0$ , then there are two distinct real roots.

b)  $3x^2 + 4x + \frac{4}{3} = 0$   
 $a=3$   $b=4$   $c=\frac{4}{3}$

$$b^2 - 4ac$$

$$(4)^2 - 4\left(\frac{1}{3}\right)\left(\frac{4}{3}\right) = 16 - 16 = 0$$

if  $b^2 - 4ac = 0$ , then there is one real root.

c)  $2x^2 - 8x = -9$   $2x^2 - 8x + 9 = 0$   
 $a=2$   $b=-8$   $c=9$

$$b^2 - 4ac$$

$$(-8)^2 - 4(2)(9) = 64 - 72 = -8$$

if  $b^2 - 4ac < 0$ , then there are no real roots.

Ex 2) Use the quadratic formula to solve the quadratic equation,  $3x^2 + 5x - 2 = 0$ .

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$a=3 \quad b=5 \quad c=-2$$

$$x = \frac{-5 \pm \sqrt{5^2 - 4(3)(-2)}}{2(3)}$$

$$= \frac{-5 \pm \sqrt{25 + 24}}{6}$$

$$= \frac{-5 \pm \sqrt{49}}{6}$$

$$= \frac{-5 \pm 7}{6}$$

$$x = \frac{-5 + 7}{6}$$

$$x = \frac{2}{6}$$

$$\boxed{x = \frac{1}{3}}$$

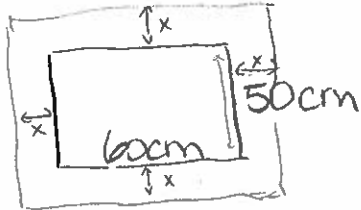
$$x = \frac{-5 - 7}{6}$$

$$x = \frac{-12}{6}$$

$$\boxed{x = -2}$$

#### 4.4 Quadratic Formula

Ex 3) Leah wants to frame an oil original painted on canvas measuring 50 cm by 60 cm. Before framing, she places the painting on a rectangular mat so that a uniform strip of the mat shows on all sides of the painting. The area of the mat is twice the area of the painting. How wide is the strip of exposed mat showing on all sides of the painting, to the nearest tenth of a centimetre?



$$A = l \times w$$

$$A_{\text{canvas}} = 60 \times 50 = 3000 \text{ cm}^2$$

$$6000 \text{ cm}^2 = (60 + 2x)(50 + 2x)$$

$$6000 = 60(50 + 2x) + 2x(50 + 2x)$$

$$6000 = 3000 + 120x + 100x + 4x^2$$

$$6000 = 3000 + 220x + 4x^2$$

$$0 = -3000 + 220x + 4x^2$$

$$4x^2 + 220x - 3000 = 0$$

$$a = 4 \quad b = 220 \quad c = -3000$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-220 \pm \sqrt{(220)^2 - 4(4)(-3000)}}{2(4)}$$

$$= \frac{-220 \pm \sqrt{48400 + 48000}}{8}$$

$$= \frac{-220 \pm \sqrt{96400}}{8}$$

$$x = \frac{-220 + \sqrt{96400}}{8}$$

$$x = \frac{-220 - \sqrt{96400}}{8}$$

$$x = 11.3 \text{ cm}$$

~~$x = -66.3$  extraneous root.~~

The width of the strip of exposed mat is 11.3 cm.